# Locality Sensitive Discriminative Dictionary Learning (ICIP 2015)

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# Outline

- Introduction
- Motivation of Our Work
- The Proposed Method
- Experimental Results
- Summarization and Future Work

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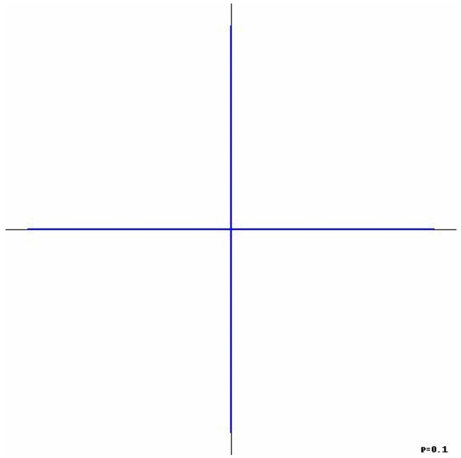
# Dictionary Learning (DL)

- A basic framework of DL:

$$\min_{D,X} \sum_{i=1}^{N} \left( \| y_i - Dx_i \|_2^2 + \tau \| x_i \|_p \right)$$

$$\min_{D,X} \|Y - DX\|_F^2 + \tau \|X\|_p$$



$\| x_i \|_p = 1$

  - Training data: $Y = [y_1, \ldots, y_N] \in \mathbb{R}^{n \times N}$
  - Dictionary: $D = [d_1, \ldots, d_K] \in \mathbb{R}^{n \times K}$
  - Coding vectors: $X = [x_1, \ldots, x_N] \in \mathbb{R}^{K \times N}$
  - Regularization parameter: $\tau \geq 0$

Emphasize representation rather than discrimination!

# Discriminative Dictionary Learning (DDL)

- A general formula of DDL:

$$\min_{\boldsymbol{D},\boldsymbol{X}} \left\|\boldsymbol{Y}-\boldsymbol{D}\boldsymbol{X}\right\|_F^2+\tau\left\|\boldsymbol{X}\right\|_p+\alpha f\left(\boldsymbol{Y},\boldsymbol{D},\boldsymbol{X},\boldsymbol{H}\right)$$

  – Structured incoherence of dictionary
  – Fisher discrimination on the dictionary and codes
  – Label consistency of codes
  – Transform-invariance of dictionary
  – Joint dictionary learning and subspace clustering
  – ……

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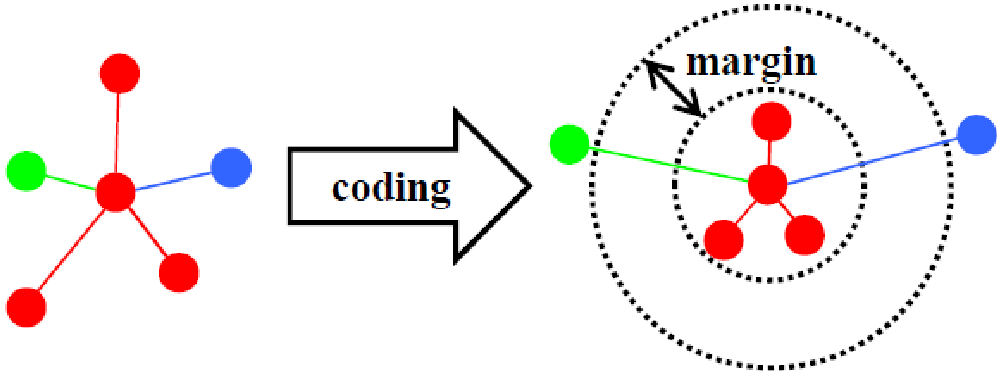
# Motivation of Our Work

- Illustration:

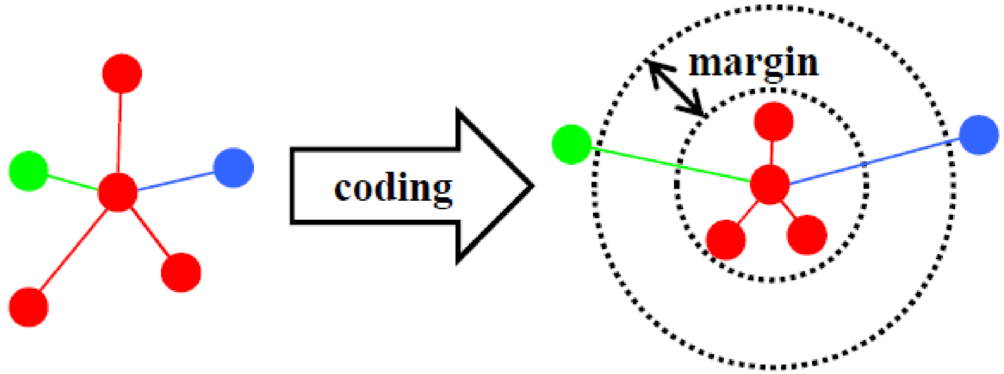



  - DDL + $k$NN classifier
  - In the coding space:
    - ➢ same-label neighbors are orderly preserved
    - ➢ neighbors with different labels are repelled

Maximize the **local** margin between different classes.

# Motivation of Our Work

- Illustration:




  – Integrate two significant characters into DL

  ➢ preserve ordinal locality

  ➢ strengthen discriminability

**L**ocality **S**ensitive **D**iscriminative **D**ictionary **L**earning (**LSDDL**)

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# The Proposed Method

- Construct two graphs via Gaussian kernel:
  - a within-class graph $G_w$
    - the corresponding weight matrix

$$\boldsymbol{W}_{w,ij} = \begin{cases} w(\boldsymbol{y}_i, \boldsymbol{y}_j), \ if\ \boldsymbol{y}_i \in N_w(\boldsymbol{y}_j)\, or\ \boldsymbol{y}_j \in N_w(\boldsymbol{y}_i) \\ 0 \qquad\qquad\qquad, \ otherwise. \end{cases}$$

  - a between-class graph $G_b$
    - the corresponding weight matrix

$$\boldsymbol{W}_{b,ij} = \begin{cases} 1 - w(\boldsymbol{y}_i, \boldsymbol{y}_j), \ if\ \boldsymbol{y}_i \in N_b(\boldsymbol{y}_j)\, or\ \boldsymbol{y}_j \in N_b(\boldsymbol{y}_i) \\ 0 \qquad\qquad\qquad\qquad, \ otherwise. \end{cases}$$

# The Proposed Method

- Determine a “good” coding process:
  - same-label neighbors are orderly preserved

$$\min_{X} \sum_{i=1}^{N} \sum_{j=1}^{N} \| \boldsymbol{x}_i - \boldsymbol{x}_j \|_2^2 \, \boldsymbol{W}_{w,ij}$$

  - neighbors with different labels are repelled

$$\max_{X} \sum_{i=1}^{N} \sum_{j=1}^{N} \| \boldsymbol{x}_i - \boldsymbol{x}_j \|_2^2 \, \boldsymbol{W}_{b,ij}$$

- Rewrite the objective:

$$\min_{X} \sum_{i=1}^{N} \sum_{j=1}^{N} \| \boldsymbol{x}_i - \boldsymbol{x}_j \|_2^2 \, (\boldsymbol{W}_{w,ij} - \lambda \boldsymbol{W}_{b,ij}) \Rightarrow \min_{\boldsymbol{X}} Tr(\boldsymbol{X}^T \boldsymbol{X} \boldsymbol{L})$$

# The Proposed Method

- The final objective function:

$$\min_{\boldsymbol{D},\boldsymbol{X}} \|\boldsymbol{Y}-\boldsymbol{D}\boldsymbol{X}\|_F^2 + \alpha Tr(\boldsymbol{X}^T\boldsymbol{X}\boldsymbol{L}) + \tau\|\boldsymbol{X}\|_F^2$$

- Alternating Optimization
  - Compute $\boldsymbol{X}$ column by column with fixed $\boldsymbol{D}$

$$\boldsymbol{x}_i^* = \arg\min_{\boldsymbol{x}_i} \| \boldsymbol{y}_i - \boldsymbol{D}\boldsymbol{x}_i \|_2^2 + \tau \| \boldsymbol{x}_i \|_2^2 + \alpha[2\boldsymbol{x}_i^T(\boldsymbol{X}\boldsymbol{L}_i) - \boldsymbol{x}_i^T\boldsymbol{x}_i\boldsymbol{L}_{ii}]$$

  - Update $\boldsymbol{D}$ with fixed $\boldsymbol{X}$

$$\boldsymbol{D}^* = \arg\min_{\boldsymbol{D}} \|\boldsymbol{Y}-\boldsymbol{D}\boldsymbol{X}\|_F^2$$

  - Alternatively minimized until convergence

# The Proposed Method

- Alternating Optimization
  - Compute $\boldsymbol{X}$ column by column with fixed $\boldsymbol{D}$

$$\boldsymbol{x}_i^* = \arg\min_{\boldsymbol{x}_i} \| \boldsymbol{y}_i - \boldsymbol{D}\boldsymbol{x}_i \|_2^2 + \tau \| \boldsymbol{x}_i \|_2^2 + \alpha[2\boldsymbol{x}_i^T(\boldsymbol{X}\boldsymbol{L}_i) - \boldsymbol{x}_i^T\boldsymbol{x}_i\boldsymbol{L}_{ii}]$$

➢ First derivative: $2\boldsymbol{D}^T\left(\boldsymbol{D}\boldsymbol{x}_i - \boldsymbol{y}_i\right) + 2\tau\boldsymbol{x}_i + 2\alpha\boldsymbol{X}\boldsymbol{L}_i$

➢ Second derivative: $2\left[\boldsymbol{D}^T\boldsymbol{D} + \left(\tau + \alpha\boldsymbol{L}_{ii}\right)\boldsymbol{I}\right]$

➢ The objective function is convex for $\boldsymbol{x}_i$

  - Optimal solution

$$\boldsymbol{x}_i^* = \left[\boldsymbol{D}^T\boldsymbol{D} + (\tau + \alpha\boldsymbol{L}_{ii})\boldsymbol{I}\right]^{-1}\left(\boldsymbol{D}^T\boldsymbol{y}_i - \alpha\sum_{m\neq i}\boldsymbol{x}_m\boldsymbol{L}_{mi}\right)$$

# The Proposed Method

- Alternating Optimization
  - Update $\boldsymbol{D}$ with fixed $\boldsymbol{X}$

$$\boldsymbol{D}^* = \arg\min_{\boldsymbol{D}} \|\boldsymbol{Y} - \boldsymbol{D}\boldsymbol{X}\|_F^2$$

➢ First derivative: $2(\boldsymbol{D}\boldsymbol{X} - \boldsymbol{Y})\boldsymbol{X}^T$

➢ Second derivative: $2\left[\boldsymbol{I} \otimes (\boldsymbol{X}\boldsymbol{X}^T)\right]$

➢ The objective function is convex for $\boldsymbol{D}$

  - Optimal solution

$$\boldsymbol{D}^* = \boldsymbol{Y}\boldsymbol{X}^T\left(\boldsymbol{X}\boldsymbol{X}^T + \eta\boldsymbol{I}\right)^{-1}$$

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# Experimental Setup

- Datasets and Features
  - Extended YaleB: 504-dimensional random-face features
  - AR: 540-dimensional random-face features
  - Caltech 101: 3000-dimensional BoVW+SPM features
- Comparing Algorithms
  - the baseline support vector machine (SVM)
  - the classical SRC [PAMI 2009] and CRC [ICCV 2011]
  - the novel locality-sensitive SRC (LSRC) [PR 2013]
  - the other famous DL methods: DLSI [CVPR 2010], FDDL [ICCV 2011], LC-KSVD [PAMI 2013], DDL-PC [ACCV 2012]
  - the recently proposed LPDDL [ICIP 2014]

# Results

Classification accuracies (%) on three databases.

| | Extended YaleB | AR | Caltech101 |
|---|---|---|---|
| SVM | 95.6 | 96.5 | 64.6 |
| SRC | 96.5 | 97.5 | 70.7 |
| CRC | 97.0 | 98.0 | 68.2 |
| LSRC | 95.7 | 97.4 | 73.4 |
| DLSI | 97.0 | 97.5 | 73.1 |
| FDDL | 96.7 | 97.5 | 73.2 |
| LC-KSVD | 96.7 | 97.8 | 73.6 |
| DDL-PC | 95.3 | 96.0 | 73.2 |
| LPDDL | 96.4 | 97.3 | 73.3 |
| **Ours** | **97.0** | **98.0** | **73.6** |

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# Summarization

- Conclusion
  - Performance is competitive with previous arts.
  - The locality sensitive objective function is useful to DDL + $k$NN classifier.

- Main Contribution
  - Preserve local relationship of same-label points and induce a margin between points from different classes.
  - Utilize analytical solutions in both dictionary learning and coding phases.

# Future Work

- Reducing the consumed time for cross-validation in the training phase.
- Generalize our work to analysis dictionary learning framework:

$$
\begin{aligned}
\min_{\mathbf{\Omega},\mathbf{X}} \quad & \|\mathbf{X} - \mathbf{\Omega}\mathbf{Y}\|_F^2 \\
s.t. \quad & \|\mathbf{X}\|_0 \leq T_0, \\
& \|\omega_i\|_2 = 1,\ i = 1, 2, \cdots
\end{aligned}
$$

R. Rubinstein, A. M. Bruckstein, and M. Elad. Dictionaries for sparse representation modeling. *Proc. IEEE*, 98(6):1045-1057, Jun. 2010.

# Thank you!

## Questions please?